Technical Correspondence

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MICROSTRIPLINE DESIGN CALCULATIONS

☐ A letter from David L. Brook informed me that values obtained with Eqs 49 and 50 on page 5-34 of *The ARRL UHF/Microwave Experimenter's Manual* do not match the values in Tables 5-10 that follow the equations. I found that the two equations are incorrect. Here are the corrections:

$$Z_0 \approx \frac{120\pi}{\left(\frac{W}{h}+1\right)\sqrt{\epsilon_r+\sqrt{\epsilon_r}}} \quad \text{(Eq 49)}$$

$$\frac{W}{h} \approx \frac{120\pi}{Z_0\sqrt{\epsilon_r+\sqrt{\epsilon_r}}} - 1 \quad \text{(Eq 50)}$$

The author's text explains that the equations present only a first approximation of appropriate line width. Fisk's *Ham Radio* article[1] (which is mentioned with Eq 49) states that the equations are accurate within a few percent when $\epsilon_r > 4.0$, and, when $0.2 < W/h < 10$. Since readers may use the equations for their own designs, it may be handy to estimate the quality of the approximation.

The most widely accepted microstripline equations come from H. Sobol.[2] They appear in Motorola application note AN-548A and many other sources. (The equation numbers here match those of the Motorola note.) Once a ballpark line width has been determined with Eq 50, derive an accurate line width using an iterative approach and the following equations:

$$Z_0 \approx \frac{377h}{\sqrt{\epsilon_r}\,W_{eff}\left[1+1.735\,(\epsilon_r)^{-0.0724}\left(\frac{W_{eff}}{h}\right)^{-0.836}\right]} \quad \text{(Eq 1)}$$

Where W_{eff} is the effective line width:

$$W_{eff} = W + \frac{t}{\pi}\left[\ln\left(\frac{2h}{t}\right)+1\right] \quad \text{(Eq 3)}$$

and

- h = dielectric thickness
- ϵ_r = dielectric constant
- t = conductor thickness
- Z_0 = line characteristic impedance

In the course of my work, I also checked Eq 51 in the *ARRL UHF/Microwave Experimenter's Manual,* which estimates the velocity factor of a microstripline. That equation does not consider effective line width, and its results vary from those shown in Tables 5-10 by a few percent. The tables closely match the velocity factor given by Sobol's equations:

$$k = \sqrt{\frac{\epsilon_r}{1+0.63\,(\epsilon_r-1)\left(\frac{W_{eff}}{h}\right)^{a}}} \quad \text{(Eq 8)}$$

where

$a = 0.1225$ when $W/h \geq 0.6$, 0.0297 when $W/h < 0.6$

[1]J. Fisk, "simple formula for microstrip impedance," the ham notebook, *Ham Radio*, Dec 1977, pp 72-73.
[2]H. Sobol, "Extending IC Technology to Microwave Equipment," *Electronics*, Mar 20, 1967, p 112.

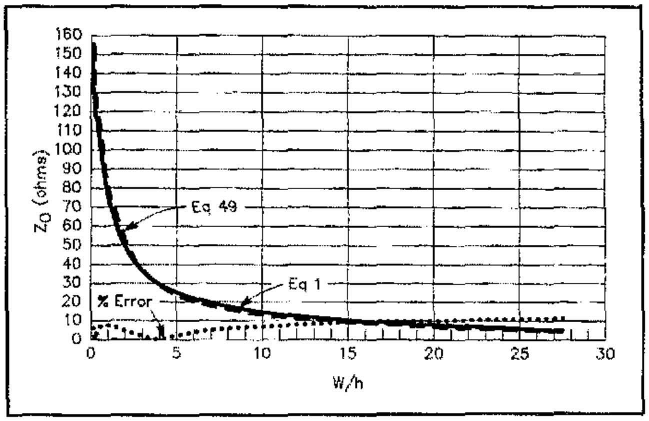

Fig 1—A plot of line width calculated with Eq 50, actual line impedance calculated from Eq 1 and the error percentage between the two (error = Eq 50/Eq 1). This plot is for 1/16-inch fiberglass-epoxy PC board with 1-oz copper coating on both sides.

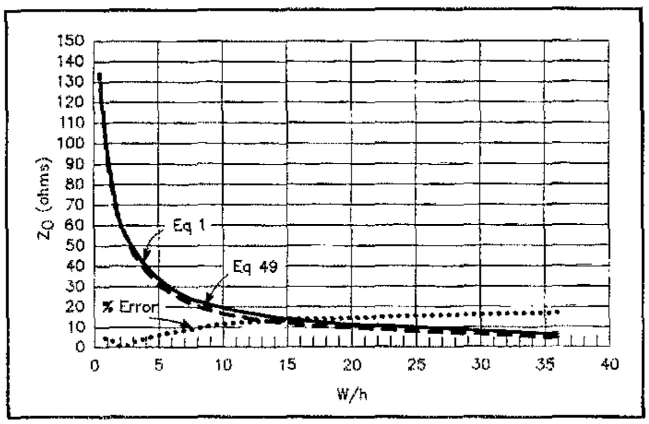

Fig 2—A plot of line width calculated with Eq 50, actual line impedance calculated from Eq 1 and the error percentage between the two (error = Eq 50/Eq 1). This plot is for 1/16-inch Teflon-fiberglass PC board with 1-oz copper coating on both sides.

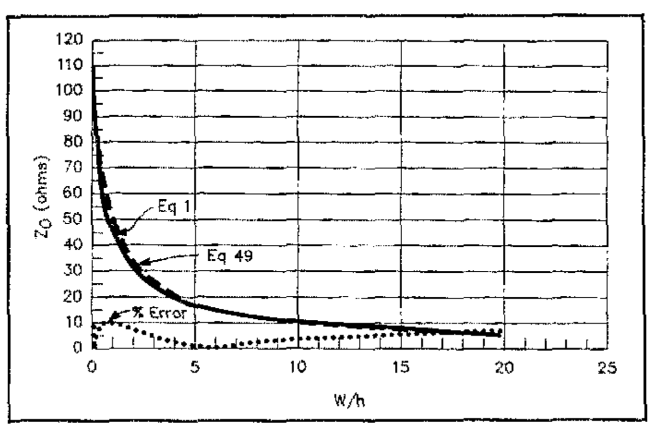

Fig 3—A plot of line width calculated with Eq 50, actual line impedance calculated from Eq 1 and the error percentage between the two (error = Eq 50/Eq 1). This plot is for 0.050-inch Epsilam-10 PC board with 1-oz copper coating on both sides.

$$VF = \frac{k}{\sqrt{\epsilon_r}} \quad \text{(Eq A)}$$

While checking my work, I ran off some new tables that show Z calculated with Eq 1, and the percentage difference between the target and the calculated Z. They also show velocity factors and wavelengths based on Sobol's equations. The new tables illustrate the quality of the approximation given by Eqs 49 and 50: The maximum error is about 12% when $\epsilon_r > 4.0$ and about 17% for $\epsilon_r = 2.55$. Figs 1 through 3 are plots of line width calculated with Eq 50, actual line impedance calculated from Eq 1, and the error percentage between the two (error = Eq 50/Eq 1). The three plots illustrate the situation for three values of ϵ_r.

Interested readers can get a copy of the revised tables by sending a business-size SASE to the Technical Department Secretary, 225 Main St, Newington, CT 06111; request the MWB TABLES 5-10 TEMPLATE.—*Bob Schetgen, KU7G, ARRL Staff*

Author H. Paul Shuch responds:

☐ To mix metaphors, if Jim Fisk were alive today, he'd be turning over in his grave. I regret the editing errors which caused his microstrip synthesis and analysis equations (*UHF/Microwave Experimenter's Manual*, Chapter 5, Eq 49 and Eq 50) to be cited in error, and I'm sorry I failed to catch the error in time to correct the second printing of the book. I've just double-checked my original manuscript, and the correct equations are indeed as Bob has stated.

I must, however, take minor exception to Bob's claim that Sobol's equations are "the most widely accepted microstripline equations." I mean no disrespect to Hal Sobol, whose work I admire (all the more so since I had the opportunity to meet him in person some years ago). I cited Sobol's equations in an article I published in 1978[3] as my Reference #27 in Chapter 5, and in fact, I published a computer algorithm for an iterative solution to his synthesis and analysis equations as an Application Note in 1976.[4] But, as Hal Sobol would be the first to admit, there *are* other popular microstrip dimensioning tools!

To the best of my knowledge, all of the authors I cite in my references 20 through 31 applied curve-fitting techniques to the generation of their microstripline equations. That is, they measured the properties of various microstriplines, and then utilized multivariate statistical tools to come up with a likely set of equations to describe what they observed. Thus, Sobol's equations are approximations, as are those of Hal Wheeler, Seymour Cohn, Maurice Schneider, Bryant and Weiss, Yamashita and Mittra, and—yes—even Jim Fisk. Sobol's approximation is an excellent one, to be sure. But that does not make it the standard against which all other algorithms are to be measured.

The correct way to determine the "error" in the Fisk approximation is *not* to compare its mathematical results to those obtained utilizing Sobol's, or anyone else's equations. Rather, build a bunch of microstriplines of different dimensions, on different substrates. *Measure* their characteristic impedance and propagation velocity in the laboratory. Then, *compare* these observations to those values which the equations predict. In so doing, I think you'll find that Fisk's approximation, like Sobol's, exhibits workable accuracy.

There was also a significant typographical error in my Equation 51. The radical on the right side of the equation is shown extending above the *0.5* in the numerator; it should not. The correct equation is:

$$\left(\frac{1}{V_r}\right)^2 = 1 + \left[(\epsilon_r - 1)\left(0.5 + \frac{0.5}{\sqrt{1 + \frac{10h}{W}}}\right)\right] \quad \text{(Eq 51)}$$

[3]H. P. Shuch, "Microstrip—magical PC technique explained," *73*, Oct 1978, pp 80-87.

[4]H. P. Shuch, *Microstripline Pack*. HP-25 Programs, Microcomm Application Notes, San Jose, CA, revised April 1976 (out of print).

Despite assertions to the contrary, this equation does indeed consider fringing effects. It simply doesn't use the expression effective line width, W_{eff}, as Sobol's equations do. To understand the relationship, let's derive Equation 51 together.

For a wave propagating through a dielectric medium, the relative wave velocity (with respect to the speed of light in free space) varies inversely as the square root of the dielectric constant, or relative permittivity of the dielectric medium. Thus:

$$V_r = 1 / (\epsilon_r)^{1/2} \quad \text{(Eq B)}$$

But that's for a wave completely enclosed in the dielectric medium. In the case of microstripline, the wave is half in the medium, and half in air (an effect we call "fringing"), so we have to modify the above equation. One way to do so is to consider the *effective* permittivity of the air-dielectric combination, which I'll call ϵ_{eff}:

$$V_r = 1 / (\epsilon_{eff})^{1/2} \quad \text{(Eq C)}$$

Now solving for ϵ_{eff} gives us:

$$(1 / V_r)^2 = \epsilon_{eff} \quad \text{(Eq D)}$$

and you should recognize where the left-hand side of Equation 51 came from. For the right-hand side, we need to come up with an approximation of ϵ_{eff}.

If the effective dielectric constant for microstripline were independent of strip width (it isn't), then our approximation would be easy (it's not). Consider that half the wave is in air (relative permittivity = 1). This should make effective permittivity equal to simply the permittivity of the dielectric material, divided by two.

Unfortunately, fringing varies with line width (the effect which Sobol captured in his "effective line width" factor), so the approximation will be more complex than this. The right-hand side of Equation 51 is derived from the following approximation of effective dielectric constant:

$$\epsilon_{eff} \approx 1 + \left[(\epsilon_r - 1)\left(1 + \frac{1}{\sqrt{1 + \frac{10\,h}{W}}}\right) / 2\right] \quad \text{(Eq E)}$$

The third factor within the brackets above (the "divide by two") is equivalent to our dividing permittivity by two, in our simple line-width-invariant model above. Now if we apply that factor to the one which precedes it, we get:

$$\epsilon_{eff} \approx 1 + \left[(\epsilon_r - 1)\left(0.5 + \frac{0.5}{\sqrt{1 + \frac{10\,h}{W}}}\right)\right] \quad \text{(Eq 51)}$$

which is what we *should have had* for Equation 51 in the book.

The fact that "tables [5 through 10] do closely match the velocity factor given by Sobol's equations" should serve to verify that the above approximation takes into account all significant determinants of effective line width.

Suppose now that the maximum error in Fisk's equations really is about 12%. Is this a significant error? Consider that a 12% impedance mismatch results in an SWR of 1.12, or a reflection loss contribution of about two hundredths of a decibel. Is that really enough loss to quibble over? I suspect that etching errors will be more significant than modeling errors, no matter whose equations you use.

At this point, an additional word about Hal Sobol might be in order. My only personal contact with him, to which I alluded earlier, was at the 1977 IEEE International Microwave Symposium in San Diego. I had just presented a paper at which, among other things, I introduced an entirely new topology for a microstrip bandpass filter. After my talk, I was greeted rather brusquely by a gentlemen I knew only by reputation, who stated "Your filter can't possibly work." I inquired why, rather defensively, and was told, "It's too simple!"

Hal Sobol appreciated simplicity in engineering solutions. I strongly suspect that he would have appreciated Jim Fisk's microstripline equations. After all, they merely did what Sobol's own approximations purported to do, only more simply. —*H. Paul Shuch, N6TX, Professor of Electronics, Pennsylvania College of Technology, One College Ave, Williamsport, PA 17701*

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Feedback

CORRECTIONS TO ACCURATE SINGLE-LAYER-SOLENOID INDUCTANCE CALCULATIONS

☐ There are a few pecadillos in my April *QST* Technical Correspondence article[1] that I'd like to get corrected as soon as possible. The errors are all mine and I should have caught them.

Please make the following corrections:

• Under Equation 2, replace "p = coil pitch" with "p = coil pitch [b/N]."

This is the way they defined it in the older days, apparently.

• Replace "d = coil diameter (wire center to wire center)" with "d = wire diameter." Remove the original parenthetical expression.

• In the second column, the sentence "Nagoka's constant for short coils (d/l less than one) is:" should read "Nagoka's constant for short coils (2a/b greater than one) is:"

• Below Equation 5, replace "B = coil diameter/coil length" with "B = coil length/mean coil diameter (2a)."

• In the last sentence in column 2, the parenthetical expression "(d/l greater than one)" should read "(2a/b less than one)."

• The correct value for Wheeler's approximation in Fig 1C is 163.265, not 132.7 nH.

• Equations 7 and 8 both require a negative exponent. The curve fit is good, but accuracy suffers with values in the region of one to two. Use Equation 7 when 2a/b is greater than 2; use Equation 8 when 2a/b is less than 2.

$$K_s = 0.9694 \left[\frac{2a}{b}\right]^{-0.6932} \quad \text{(Eq 7)}$$

$$K_L = 0.9617e^{\,0.2913(2a/b)} \quad \text{(Eq 8)}$$

• I have provided Table 1 which contains the actual values one needs to check out the equations. The graphs supplied in the original article are incorrect and have insufficient resolution to verify results.

Table 1

Inductance Values for Short, Equal Length/Diameter and Long Coils

Short coils—Winding Diameter = 0.5, b = 0.25, TPI = 16, N = 4

Wire Size	*Inductance (nH)*
16	224.7
18	222.02
20	228.5
22	231.96
24	236.17
26	241.0
28	246.35
30	252.03

(Simple approximation yields 210.53 nH.)

Equal Length/Diameter coils—Winding Diam = 0.5, b = 0.5, TPI = 8, N = 4

Wire Size	*Inductance (nH)*
14	163.9
16	165.57
18	168.32
20	171.93
22	176.80
24	181.22
26	186.61
28	192.38
30	198.45

(Simple approximation yields 137.93 nH.)

Long coils—Winding Diam—0.25, b = 0.5, TPI = 16, N = 8

Wire Size	*Inductance (nH)*
16	205.56
18	200.93
20	198.79
22	198.58
24	199.91
26	202.44
28	205.95
30	210.16

(Simple approximation yields 163.267 nH.)

Here's some additional information: For the larger wire sizes, and relatively small diameters, the inductance increases because the mean coil diameter is the mandrel diameter plus the wire diameter. Therefore, the coil diameter increases more rapidly with larger wire sizes if the mandrel size is relatively small. This is important to realize if drill shanks are used as mandrels. Mandrel diameter is the same as winding diameter and not equal to 2a.

Brian Beezley, K6STI, has written a BASIC program that calculates the inductance values. There are minor differences with these results and the numbers in Table 1 due to rounding. Brian has uploaded the program (*IND.EXE*) to the ARRL BBS (203-665-0090). The only limitation to this program is that the number of coil turns is limited to 999.99.

Some correspondents, notably Tuggle, N3HZK, and Charles Michaels, W7XC, correctly point out that the true inductance of a coil can be significantly different from its effective inductance at the terminals. This is due to several other considerations necessary to the realization of a specific coil design. These are: distributed capacitance, C_o, pigtail lead inductance and capacitance, Q, grounding one end of the coil, closeness to shielding, self-resonance effects, etc. I felt that the topic of true inductance was complex enough by itself, and as such, left realization considerations as another topic for discussion.

Several respondents have presumed that since the formulations were generated at the turn of the century that they are only applicable to frequencies in the tens of kilohertz region. This is not so. There are no frequency-dependent terms used in the calculation of inductance unless the wire is made of magnetic material. Then, these effects for the most part, become negligible above 500 kHz.

I hope this clarifies any questions readers have, and I apologize for any inconvenience this may have caused.—*Hank Meyer, W6GGV, 3801 Walnut Ave, Long Beach, CA 90807*

[1]H. Meyer, "Accurate Single-Layer-Solenoid Inductance Calculations," Technical Correspondence, *QST*, Apr 1992, pp 76-77.