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ORBITAL ANALYSIS BY SLEIGHT OF HAND

A review of manual, intuitive satellite tracking techniques

Modern digital computers, along with low-cost, high-power software, have been indispensable tools in the quest to communicate via orbiting satellite. Tracking software has become so ubiquitous that a whole generation of enthusiasts is unaware that there once was another way to track the early OSCARs. While I'm not opposed to progress, I am of the opinion that the magical computer tends to separate its user from the basics of the underlying application. Through a review of manual, intuitive satellite tracking techniques, we can gain a more thorough understanding of the rather simple forces of nature underlying those mystical Keplerian Elements.

Introduction

There is a curious object on display in my classroom—a pancake-flat Plexiglas™ disk about a half meter in diameter. A North polar projection map of the earth appears on one side of the disk, and a South polar projection appears on the other. Sandwiching this pre-Columbian globe are two transparent overlays, pinned to the poles with a robust grommet so as to rotate freely. On the rotating panels, great arcing lines with cryptic notations are inscribed in grease pencil. The object looks very much its part—an icon to the high priest of satellite navigation.

You probably know this device as an Oscar-Locator, a graphical aid to satellite tracking. This granddaddy of all Oscar-Locators was in fact the original OSCAR 1 orbital plotting board, preserved throughout these countless generations of amateur radio satellites. We called it a Satelabe then—a word derived from Astrolabe—the navigational instrument that guided explorers across un-charted oceans from ancient times until the development of the sextant in the 18th Century. Even though my students would relegate it, along with my Osborne 1 computer and log-log-decitrig slide rule, to the musty, dusty shelves of a technology museum, I keep it for the most utilitarian of reasons—because it still works.

The early days

The microcomputer is today as indispensable a part of the world of satellite communications as the mini-HT is to amateur radio. Starting with Dr. Tom Clark's legendary *BASIC Orbits* program and continuing up through the present software array with its dazzling high-resolution graphics, we all have tools at our disposal, the likes of which NASA could only dream about during the days of Apollo. Yet in those eras of antiquity, BC (Before the Computer), a handful of dedicated visionaries managed to conceive, construct, and connive into orbit, the world's first non-Government sponsored artificial Earth satellite. They did so with tools that today would be considered laughably crude, but they did it. They left us AMSAT as their legacy. They left us their Satelabe, as a reminder.

I was not really one of the original OSCAR cadre, although I sat at their feet in awe. As a high school student sitting quietly in the back of the room, watching my heroes concoct their minor miracle, I said to myself: "Some day, when I grow up, this is what I want to do." And someday, when I grow up, I will.*

In the meantime, I've been privileged to teach satellite communications to a whole generation of technologists. Only their way, it's hard to tell where the computer ends and the technician begins. Armed with their silicon ephemera, they manipulate Keplerian Elements to the twelfth significant figure, produce orbital predictions in double precision, and haven't the foggiest idea what they all mean.

An old tool

That's where the Satelabe comes in. By using this old tool, and applying mathematical concepts no more advanced than those available to the ancients, one can visualize the balance of forces that hold a satellite in its orbit. With purely manual techniques, one can perform orbital analysis to far less accuracy, but with far greater clarity than can be accomplished by digital computer.

That's also what this article is all about. In it, I'll review a few of the basics of the satellite orbit, armed with nothing but a pencil and a pocket calculator (a few years ago, I would have said slide rule). Although I doubt I'll ever wean you from the MegaTrak 1000 program running on your 100-MHz 80686 with the gigabyte hard drive, I hope to remind those of you who might have forgotten, what's really going on behind the zillion-pixel screen, back in the land where the ones and zeros cavort in wild abandon.

Circles and ellipses: an eccentric view

A good place to start our description of the orbits of communications satellites is with a review of the most historically significant controversy in astronomy—the nature of our solar system. Probably the first to propose a heliocentric theory of the universe was the Greek astronomer Aristarchus of Samos (circa 310-230 BCE). His view received little attention from the ancients, who favored a geocentric scheme, as later formalized by the Greco-Egyptian mathematician Claudius Ptolemaeus in the 2nd century CE. In Ptolemy's system, simple circular motion was used to describe the motion of all visible celestial bodies around the earth. The apparent retrograde motion of the planets was explained by a rather complicated system of epicycles on deferents.** Still, predictions of celestial events based upon the Ptolemaic system were crude at best.

The great Polish astronomer Nicholas Copernicus may well have been trying to smooth out some of the inconsistencies in the Ptolemaic system's prediction of eclipses. He was probably the first astronomer since Aristarchus to propose anew a heliocentric solar system (*De Revlutionibus Orbius Coelestium*, 1543). Although he placed the Sun in its rightful place, his system still retained a variant on epicycles to explain retrograde motion. Circular orbits, it seemed, were tricky things.

Tycho Brahe was a renowned fence sitter. Unable to decide the relative merits of the Ptolemic and Copernican systems for himself, he created one of his own, incorporating elements of each. This Danish astronomer saw an immobile earth around which the sun revolved, with the other five known planets then revolving around the Sun. To support his model, he recorded a lifetime of observations of the planets, the Moon, and Supernova 1572. His excellent observations, published posthumously by Kepler (in *Tabulae Rudolphinae*, 1627) led to the breakthrough that eliminated the inconsistencies in all earlier solar system models. Indirectly, it also gave us our most powerful tools for satellite orbital analysis—the Keplerian Elements.

Professor of mathematics at Graz, Johannes Kepler proposed in 1609 (in *Astronomia Nova*) that planetary motion could be described not by circles, but by ellipses. He showed that any planet (and by extrapolation, satellite) must orbit in an ellipse with its primary at one focus. His laws of motion further described the change of orbital velocity required throughout a stable elliptical orbit, and explained (*Harmonice Mundi*, 1619) the interrelationship between the size of the ellipse and the orbital period. Kepler's elliptical orbits made possible a full understanding of not only the heliocentric solar system, but three and a half centuries later, the orbit of your favorite OSCAR.

It is important to note here that a circle is simply a special case of an ellipse, one whose eccentricity equals zero.** Since our earliest artificial earth satellites (Sputnik I, OSCAR I, etc.) all had low eccentricities, we tended to analyze their orbits as circular. That's where the early OscarLocator came in. It turns out that the motion (relative to an observer on the ground) of

*Okay, I exaggerate my modesty slightly. I have had a minor role in all of this, rising through the ranks of Project OSCAR volunteers to eventually become Technical Director, and then Chairman of the Board, a position I held until 1990.

**To visualize this rather convoluted scheme, think of the popular carnival ride called the Tilt-a-Whirl. Each rider (planet) sits on a cart. The cart sits on a platform that describes a circular orbit (deferent), while whipping around in its own circular sub-orbit (epicycle).

**Eccentricity is a measure of the "lopsidedness" of any ellipse, on a range of 0 to 1. Values approaching 0 represent a round, and those approaching 1 represent a flat shape.

a satellite in circular orbit can be easily described as an arc on a polar projection map. The Satelabe is such an arc on a map. With it, we can not only extrapolate any orbit into the future for prediction of Acquisition of Signal (AOS) and Loss of Signal (LOS) at a ground location, but we can also project required antenna bearings (azimuth and elevation) for a given pass, and predict mutual satellite visibility for two ground stations. Useful, no?

More complicated orbits (such as the highly eccentric Phase III and Molniya satellites) defy such simple graphical analysis because their ground tracks tend to corkscrew, and do not necessarily repeat over time. It is for such orbits that tracking software really shines. However, we still have a good number of useful satellites in nearly circular orbits (PACSATS, MicroSats, MIR, space shuttles, various weather satellites, and of course the most useful Clarke, or geosynchronous, orbit). All but the latter (which stands motionless in the sky, from our terrestrial vantage point) can benefit from Satelabe analysis.

The ideal orbit

We will begin our analysis of satellite orbits by making quite a few simplifying assumptions. For starters, let's consider an artificial satellite in perfectly circular orbit (that is, eccentricity of zero) around a perfectly spherical earth of uniform density, with no atmosphere. Of course, our planet is both lumpy and oblate (wider at the equator than at the poles, by about 0.1 percent or so); however, ignoring these details simplifies orbital analysis. We will correct for reality later. Similarly, let's turn an n-body problem (one which includes the gravitational effects of the moon, sun, planets, and stars) into a much simpler 2-body problem, by pretending that nothing exists but our satellite orbiting such a perfect planet.

For this simple and ideal case, only two influences determine the motion of the satellite around its primary: the force of gravity (pulling the satellite toward Earth), and the pseudo-force of the inertia (pulling the satellite away from Earth). Further, for a stable orbit (one that does not change over time), these two forces must be balanced, in exact equilibrium. Because we can define both gravity and inertia mathematically, and set the two equal to each other, we produce what I call the Basic Orbital Equation. I'll spare you the derivation (it appears in the literature) and cut to the result:

$$\frac{mV^2}{r} = \frac{GMm}{r^2} \qquad \textbf{(1)}$$

In **Equation 1**, little **m** stands for the satellite's mass, big **M** for the mass of the earth, **V** for the satellite's velocity, **r** for the radius of its orbit, and **G** for Newton's universal gravitational constant. The left-hand expression relates to inertia, and the right-hand side to gravity. As I said, a stable orbit requires that the two be equal. What's interesting about this equation is that it can be simplified. The **r** in the left-hand denominator can cancel one of the **r**'s in the right. The **m**'s in the two numerators can cancel. This leaves us with:

$$V^2 = \frac{GM}{r} \qquad \textbf{(2)}$$

which is not only simpler, but allows us to draw some interesting conclusions. For one thing, you'll see that **Equation 2** makes no reference at all to the mass of the satellite. The orbital behavior of any satellite appears *independent* of its mass!* Periodically, my students try to sell me on the notion that the mass of the satellite is absent from **Equation 2** because it's negligible relative to that of the earth, thus can be ignored. Negligible though it may be, this is not the reason. You can see from the above that, when equating inertia to gravity, the satellite's mass *cancels.*

The next interesting thing we can learn from this equation is that orbital radius varies inversely with the *square* of velocity. So you see that as you move a satellite closer to earth, it moves faster. And (because the relationship is a square), if you place it really close to earth, it moves *really* fast. This is consistent with our observations. The space shuttle, only a couple of hundred kilometers up, zips right along, orbiting our planet in about an hour and a half. Our natural satellite, the Moon, is closer to 400,000 km up. It meanders across the sky, taking a whole moonth—er, month—to orbit. Also, at an intermediate distance of 36,000 km, satellites in the so-called Clarke** or geosynchronous orbit move at moderate speeds, orbiting the Earth in exactly 24 hours.

Notice now what appears in the numerator at the right of **Equation 2**. Contrary to popular belief, the **GM** product is not a Chevy. Rather, it is a constant for all satellites orbiting the earth. If we know this value (and we do: 4 x 10^{14} m^2/s^3), we can easily calculate the velocity for a satellite orbiting at any radius, or the orbital radial that would correspond to any velocity. Try this, for example: What is the

*The *launch* of a satellite into its given orbit, on the other hand, is highly mass dependent. The more massive the satellite, the more thrust is necessary to insert it into its orbit. You have to kick the football hard, so it will not only sail *high* enough to clear the goal post, but will also be moving *fast* enough not to fall short of the goal line. That is why, for example, the launch for a heavy Phase III D is so much more costly than that for a MicroSat. Did you ever wonder how we kick a Phase III football into its intended orbit? Why with an Apogee Kick Motor, of course!

**Named for Arthur Charles Clarke, communications engineer and science fiction author who first proposed this orbit in a 1945 *Wireless World* article.

fastest possible velocity for a satellite orbiting our ideal earth?

Well, we know from **Equation 2** that the lower the orbit, the faster the velocity. So, what is the lowest possible orbital radius? If we ignore atmospheric drag, trees, mountains, and tall buildings, and rule out subterranean orbits (the tunnel isn't finished yet), that would be the radius of the Earth—about 6370 km. Plugging this **r**, and the **GM** product, into **Equation 2**, we come up with a speed of about 7900 meters per second, or 17,700 mph. So now we know the top speed for your satellite, or my motorcycle.

But what about Evel Knievel's motorcycle? Rumor has it his rocket-powered monster can exceed the speed we just computed! What will happen to him if it does? Maximum orbital velocity is also called escape velocity. Anything that exceeds this speed will break free of earth's gravity (or more properly, will find inertia exceeding the force of gravity). So, in excess of 17,700 mph, Mr. Knievel will find himself flying free of Earth's influence, departing into outer space. (Some say he's already there.) In fact, any interplanetary spacecraft must be accelerated to beyond escape velocity.

Our next task is to determine orbital period—the time required for our satellite to circumnavigate the Earth exactly once. If we know orbital radius, we also know circumference, or the distance the satellite travels in one orbit. Because a circle contains 2π radians, the distance traveled in one orbit equals $\mathbf{2\pi R}$. At the velocity calculated with **Equation 2**, orbital period is: $\mathbf{t = d/v}$, so:

$$t = 2\pi \, (r^3 / GM)^{1/2} \qquad \textbf{(3)}$$

We can now do what Clarke did 50 years ago—compute the required orbital altitude to achieve geosynchronicity. This requires us to rearrange **Equation 3** to solve for **r**—an operation that is a bit awkward, but not an algebraic impossibility. Let's see now, if I did this right:

$$r = (GM \, t^2 / 4\pi^2)^{1/3} \qquad \textbf{(4)}$$

Plugging in the orbital period required to synchronize with the Earth's rotation (24 hours, or 86,400 seconds), and our old friend the Chevy—er, **GM** product, the above gives us an orbital radius of about 42,290 km. Subtracting the Earth's 6370 km radius leaves us with a satellite 35,920 km above our planet. Within the constraint of our simplifying assumptions and round-off errors, that's exactly where Clarke said it should be.

Orbital increment defined

Let's return for a minute to this business of a spinning Earth. The Earth's rotation means that if a satellite orbits our planet exactly once, it will *not* necessarily come back to rest above the same point on Earth from which it started. Consider, for example, a polar orbit with a two-hour period. Say our satellite crosses the equator northbound (at a right angle), zips over the North Pole, crosses the equator southbound (again at a right angle), slips under the South Pole, and then returns northbound to the equator again. While all this has been happening, the earth has been spinning eastward, about 1000 mph worth at the equator. Now, if we note the point on the Earth over which the first northbound equator crossing took place (we call this point the orbit's *ascending node*), and then note where the next ascending node occurs, the second ascending node point is going to be about 2000 miles west of the first.

Incidentally, the vector sum of those two circular motions (the satellite's orbit and the Earth's rotation) describes a sinusoid. This is the source of those sine-wavy ground track lines on the flat map at Mission Control, which you've seen on the TV news. Now, the distance along the equator between two successive ascending notes is called *orbital increment*, or westward progression, and is measured, not in miles, but in degrees of longitude on that same flat map in Houston. Since our planet spins 360 degrees every 24 hours (more or less), the earth has spun about two twenty-fourths of 360 degrees, or about 30 degrees, during the course of a two-hour orbit. This means the two successive ascending nodes are 30 degrees apart; the orbital increment is 30 degrees.

But wait, there's an easier way. If we continue to consider our ideal, two-body problem, ignore the motion of the earth and its retinue of satellites about the sun, and accept our simplified view of a perfectly spherical earth of uniform density, then orbital increment is simple to estimate. Increment (in degrees) will equal exactly the orbital period (in minutes) divided by four! This can be readily proven by dimensional analysis, but trust me.

With the Satelabe, we use orbital increment to plot successive orbits. Say we know the equatorial crossing longitude in the ascending node for a given orbit. We lay the edge of our OscarLocator cursor over that longitude on the plotter, and the curved cursor shows the path the spacecraft will take over the Earth for the succeeding half orbit. To forecast the next orbit, we estimate increment by dividing period by four, add increment to the previous crossing longitude, move the cursor, and start again. An example: During a recent space shuttle mission, the orbital period was exactly 1 hour, 30.4 minutes. That's 90.4 minutes, which, divided by four, gives us an orbital increment of 22.6

degrees. On one orbit of that mission, the shuttle crossed the equator northbound at a longitude of 72.6 degrees west, at exactly 2145:30 Zulu. At exactly 2315:54 Zulu, 90.4 minutes later, the shuttle would cross the equator northbound again, at a latitude of 72.6 degrees + 22.6 degrees of increment, which equals about 95.2 degrees west.

How far up was that space shuttle? An orbital period of 90.4 minutes equates to 5424 seconds. From **Equation 4**, the orbital radius equals 6680 km. Subtracting the Earth's 6370 km radius, we see that the shuttle is only 310 km up. No wonder the overhead SAREX signals are so strong!

We still occasionally find equatorial crossing times for various ham satellites published in the AMSAT literature. If you know what ascending node value will bring the satellite overhead at your location, you can extrapolate from any given crossing time using the above method to estimate your next AOS.

And now, reality intrudes

The above computations work well for our ideal, circular orbit; but what happens when we consider the more general case of Kepler's famous ellipse? Even in elliptical orbit, gravity and inertia must always be in equilibrium. Because the distance between the satellite and its primary varies along the elliptical path, the force of gravity is ever changing. This requires a like change in inertia throughout the orbit, which is only possible if the satellite *speeds up and slows down.*

In fact, the orbital velocity of a satellite in elliptical orbit does indeed vary, from maximum at *perigee* (the point of the orbit that brings the satellite closest to the earth) to minimum at *apogee* (the point of farthest separation between satellite and earth). We can readily compute *apogee radius* as the sum of apogee height plus the Earth's radius. We can insert this value into **Equation 2** to determine the satellite's velocity at apogee. In a similar fashion, we can use *perigee radius* to compute the satellite's velocity at perigee. The mean orbital velocity will be somewhere between those two values, although we would need to apply some calculus to determine an exact value.

Since the distance between earth and elliptical satellite is ever changing, we can't directly apply **Equation 3** to determine orbital period. Also, although calculus gives us an exact solution, here's a simple first-order approximation. Compute period per **Equation 3** for a circular orbit with radius equal to your satellite's apogee. Do the same for perigee. Your satellite's actual orbital period is roughly between these two values. For example, a satellite in *synchronous transfer orbit* has its apogee at Clarke altitudes (the resulting period, for a circular orbit, would be 24 hours), and its perigee at space shuttle altitudes (corresponding period an hour and a half). The midpoint between these two values is just under 13 hours, which comes close to transfer orbit period.

Remember our myth of a uniform spherical earth? I guess you know by now that it simply isn't true. After all, our planet is a spinning body. Four billion years of spin have made the earth oblate—wider at the equator than across the poles. This happens to people in middle age. The more we spin, the wider we get at the equator. We're not obese, just oblate.

The same is true of our neighboring planets, the sun, and the stars in general. The inertia of a spinning body slings some of its matter outward, and makes it bulge. Even if we start with a perfectly circular orbit, this satellite bulge will cause the force of gravity to vary as the satellite circles the Earth. Because inertia and gravity must balance, the satellite will speed up and slow down, making its orbit wobble. We call this orbital wobble *oblateness precession*, and we can quantify it, although you don't really want to see the equation.

Well, okay, if you insist:

$$\Theta = 9.964 * [(R_e + h) / R_e]^{(-7/2)} * [1 - e^2]^{-2} \text{ Cos } i \qquad \textbf{(5)}$$

See what I mean? The important thing at this point is not to compute oblateness precession, but to recognize its effect—which is to make the satellite slip a little to the East with every orbit. So there goes our nice, simple estimate of orbital increment, right out the window. This explains, in part, the cumulative error in extrapolating equator crossings from a satellite's orbital period. In the short term, the estimates are acceptable for communications. However over days or weeks, a new ascending node observation becomes necessary in order to obtain acceptable OscarLocator results.

The next simplification to disprove is the notion of a stationary earth. Remember that our planet is moving around the sun at a rate of (360 degrees/year)/(365.242 days/year) = 0.985673 degrees/Day. Consequently, if we performed a simplified orbital analysis based upon the assumption of a stationary earth, the satellite would accumulate a westward error of almost a degree per day, when measured with respect to the Earth's surface.

For communications purposes, we *must* describe the satellite's orbit with respect to the earth's surface—that's where we are! Unfortunately, at almost a degree per day, it doesn't take many days for our accumulated OscarLocator error to become substantial. This is why, in practice, we try to obtain a new equatorial crossing point every couple of days. The Satelabe user gets these from computer-generated orbital prediction tables, which take into account the *sidereal precession.*

Did you happen to notice that the errors from sidereal and oblateness precession accumulate in opposite directions, one toward the East and the other toward the West? This means there is a possibility that we can design an orbit where the two effects will cancel, and we can achieve a reasonable approximation of our ideal orbit. Such an orbit does indeed exist, and it's called *heliosynchronous*, or sun-synchronous.

Satellites in sun-synchronous orbit will trace a ground track that repeats from day to day. This is most useful for earth resource assessment, and is used by NASA for many of its environmental and weather satellites. However for communications satellites, the sun-synchronous orbit has a number of advantages beyond simplicity of orbital calculation. One is that the spacecraft can be placed in perpetual sunlight—a useful feature for systems that derive their electrical power from the sun. Another is that a satellite in a properly designed sun-synchronous orbit can, over time, provide communications access to all points on the surface of the Earth. This is handy in packet store-and-forward applications. AMSAT's early Phase II satellites were all in heliosynchronous orbit, as are some of the newer MicroSats.

The final orbital simplification, the two-body assumption, is the most difficult to dispel. Our solar system has been described as being composed of the Sun, Jupiter, and assorted debris. All of it, even the debris, tugs on our satellite. I know of no simple algorithm for dispensing with the gravitational effects on an Earth satellite's orbit of the Sun, Moon, and planets. The computations are so complex as to defy manual solution. Here, then, is an area where computerized orbital analysis is not only justified, but really comes into its own.

Describing the orbit: size and shape

We can't use a computer to analyze an orbit unless we have a means for describing that orbit mathematically. The standard description of orbits used in the current generation of AMSAT tracking programs is the *modified Keplerian element set*, a collection of numbers that allows us to extrapolate the satellite's motion over time. The balance of this article is devoted to describing the Keplerian elements in conceptual, rather than computational, terms.

We've already seen how we can define the "roundness" of an orbit by its *eccentricity*, abbreviated **e**, on a scale of zero to one. Eccentricity, the first of our Keplerian elements, actually describes the shape of an ellipse quite completely. For current ham satellites, orbital eccentricities range from a low of approximately 0.00076 (UOSAT OSCAR 22) to a high of 0.72 (AMSAT OSCAR 13). That's a range of three orders of magnitude, and encompasses orbits from almost perfectly circular to very highly elongated.

Of course, the number of different orbits that could share the same eccentricity is infinite. Once having described the *shape* of our orbit, we next need to quantify its *size*. This can be accomplished in several different ways. The size of a given ellipse can be described by measuring the distance across it in the longest direction (Major Axis) or in the shortest direction (Minor Axis). These two axes intersect at right angles, midway between the two foci of the ellipse. We sometimes define orbital size in terms of half these values (Semi-Major Axis or Semi-Minor Axis). Similarly, apogee and perigee radii (the sum of which equals Major Axis) will define the size of the ellipse. We could also use orbital distance, the "circumference" of the ellipse.

Another way to define the orbit's size is in terms not of dimensions, but rather time. Here we would invoke the elliptical equivalent of **Equation 3**. Because the larger the ellipse, the longer the orbital period, if we know the period and eccentricity, we can determine all the critical dimensions of the ellipse. This is *almost* how we define orbital size when using satellite tracking software, but not quite.

Consider that a sine wave can be described either by its *frequency* (in cycles per second) or, alternatively, by its *period* (in seconds per cycle). The latter is what we measure on an oscilloscope and is readily converted to the former, because the two are reciprocals. In like fashion, the reciprocal of orbital period is orbital frequency, or mean motion. As a Keplerian Element, mean motion is abbreviated **N**, and is typically measured in orbits per day. Our current crop of satellites have mean motions ranging from 2.059 orbits per day (AMSAT OSCAR 10) up to 14.69 orbits per day (UOSAT OSCAR 11). The higher the mean motion, the shorter the orbital period, and the smaller the orbit.

Eccentricity **e** and mean motion **N** adequately define the size and shape of an orbit. It now remains for us to define the orientation of the orbit in space, and the location of the satellite within the orbit for any given point in time.

Describing the orbit: roll, pitch, and yaw

Just as aircraft require three angles (roll, pitch, and yaw) to describe their orientation with respect to the Earth, so can we locate our elliptical orbit in three-dimensional space by developing three appropriate Keplerian Elements. The "roll" parameter refers to the angle between the satellite's orbit and the Earth's equator, measured at the ascending node. This angle is called inclination, is measured in degrees, and is abbreviated i. An inclination of zero degrees means the satellite's orbit always remains over the equator, with the satellite moving eastward. Clarke orbits are an example of 0 degrees inclination. Another possible equatorial orbit, with the satellite moving westward, would have a 180 degree inclination. A polar orbit has an inclination of exactly 90 degrees.

Any inclination between 0 and 90 degrees defines a *prograde orbit*. This means that the satellite's horizontal motion is in the same direction as the Earth's rotation. Any inclination between 90 and 180 degrees makes for a *retrograde orbit*, with the satellite's lateral motion opposite the direction of the Earth's rotation. Prograde orbits are easier to achieve, because the eastward rotation of the Earth gives us some free thrust at launch time. In fact, the closer to the equator we move our launch site, the more of this free thrust is available. This is why the European Space Agency maintains its launch facility at Kouru, Guiana, practically on the equator.*

A retrograde orbit, with an inclination slightly above 90 degrees, is required to achieve sun-synchronicity. This is because $\cos i$ in **Equation 5** needs to be negative to give oblateness precession its proper direction to overcome sidereal precession. As a result, launch to heliosynchronous orbit is most readily achieved from extreme northern or southern launch sites, where there is less of the Earth's eastward rotational velocity to overcome.

Currently, our most nearly polar prograde satellites, AMSAT OSCAR 21 and Radio Sputnik 12/13, have inclinations on the order of 83 degrees. Our most nearly polar retrograde satellites, OSCARs 14, 16, 17, 18, 19, 20, 22, 25, 26, and 27, all have inclinations on the order of 98 degrees. (See, I told you heliosynchronous orbits are popular.) Our most equatorial ham satellite OSCAR 10, currently has about a 27-degree inclination.

For our "pitch" parameter, we locate the orbit's perigee point in degrees with respect to the earth's equator. Consider that perigee of an orbit, like the nose of aircraft, can be oriented up or down. An *Argument of Perigee* (abbreviated **w**) of 0 to 180 degrees indicates perigee in the Southern Hemisphere, with **w** = 90 degrees meaning perigee occurred over the South Pole. Values of **w** between 180 and 360 degrees mean that perigee occurred in the Northern Hemisphere, with **w** = 270 degrees placing perigee over the North Pole.

Our reference for Argument of Perigee was the Earth's equator. However in order to define "yaw," we need an external celestial longitudinal reference. The First Point of Aries, a fixed point in space by which we define the beginning of spring, is such a reference. If we draw a line from this reference point to the center of the Earth, and another line from the center of the Earth to the orbit's ascending node (northbound equator crossing), then the angle between these lines is our third orbital attitude parameter. This Keplerian element is called *Right Ascension of the Ascending Node* (RAAN), and is generally abbreviated Ω. Of course, since our planet is both spinning and orbiting the sun, Ω is not stable, but varies over time.

The Keplerian Elements Ω, **w**, and i give us three degrees of freedom, by which we can completely describe the orientation or our elliptical orbit in three-dimensional space. This leaves us with but one problem yet to solve: Just where within that orbit is our satellite at any given moment?

Describing the orbit: location, location, and location

Before we can say where a satellite is *now*, we have to know where it was *then*. But when is *then*? We need to make an observation and clearly document exactly when that observation was made. Epoch Time, **T**, is the exact time at which a positional reading was taken. It is normally entered into tracking software as year, day of year, and fraction of a day since 0000:00 hours. Looking up **T** for an OSCAR in a recent orbital elements printout, I see a value of: 95159.69677441. This is interpreted as Year = 1995, Day = 159 (that's June 9th), and Time = (0.69677441 * 24 hours), which comes out to about 16 hours, 43 minutes, 21.3 seconds. Don't worry, the tracking software will do the conversion for you.

So much for the *when*. Now how about the *where*? At Epoch **T**, we locate the satellite in its orbit. We use perigee as our reference point. If we divide the orbit into 360 increments that are equal by *time* (not equally spaced in position), we can measure how long since perigee, on a scale of 0 to 359. It's important to note that, even though we measure this parameter on a scale of 0 to 359 (and even call the unit "degrees"), this is *not* an angular measure of the satellite's position. Rather, mean anomaly (abbreviated **M**) tells us what time fraction of an

*As the old Greek philosopher Will Rogers said, "There ain't no free launch . . . but a little extra thrust reduces the cost."

orbit has occurred since the last perigee, as of Epoch **T**. Since the other Keplerian elements give us (indirectly) orbital period, we can then compute exactly where the satellite was in space at **T**, and where it will be at any future time.

Finally, we have assumed our satellites to be traveling in a vacuum, but as you know, we live at the bottom of a gaseous ocean. This stuff (mostly nitrogen, a little oxygen, and various trace elements), which we call air, stays pretty much near the surface—but some of it does extend into space. To compensate for atmospheric drag, we often plug a figure called decay rate (n dot) into our software. This tells by how many revolutions per day our mean motion will actually change. It's usually pretty close to zero.

Conclusions

For ease of operation, there's nothing to beat a good GUI-driven, high-resolution, math coprocessor supported satellite tracking program. After all, communicating with an orbiting satellite can be equated to downing the soaring eagle with a hurled ping pong ball, while blindfolded. The computer can at the very least help to strip the blindfold away. And, yes, I admit that my lab on campus boasts a fairly capable PC, on which my students crunch ephemera using a number of AMSAT tracking programs.

However on the wall above the computer, hangs that original OSCAR I Satelabe of which I spoke earlier. It occupies a place of honor, as it were, an anachronism of inestimable value. The humble Satelabe not only helps the students (and their Professor, as well) visualize the intricacies of the elliptical orbit; but as we reach for the stars, it also serves as a reminder of where we started, how far we have come . . . and just how much further the journey may yet carry us. ■