solid-state microwave amplifier design

A complete design approach for uhf and microwave amplifiers, and the performance tradeoffs which result with simplified design methods

In two recent articles I introduced a rather simplistic design method for matching microwave transistors in microstripline preamplifiers for 1296 MHz.[1,2] The designs proved entirely acceptable for amateur applications, exhibiting input and output vswr well below 2:1, and yielding gains within 0.5 dB of those theoretically obtainable from the transistors used in the circuits. In fact, these designs were so successful that they ultimately formed the basis for a commercial product line for the 1296 MHz band. There is, however, no reason why full maximum available gain, and true 1:1 vswr, should not be achievable in microstripline amplifiers if simplistic design methods are abandoned in favor of a more complex, rigorous approach. In military and aerospace electronics, where the ultimate in performance is essential, such perfectionism has become a watchword. Small wonder, then, that a few discerning radio amateurs are attempting to achieve state-of-the-art performance in their microwave transistor designs.

This article is an exposition of state-of-the-art design techniques and is intended to be instructional, rather than constructional. By briefly reviewing the design approach applied in my previous articles, I shall attempt to identify the omissions which cause the resulting amplifier performance to depart from optimum. Following a brief discussion of transistor parameter characterization, I will show the mathematical procedures necessary to overcome the performance shortcomings of my previous designs. This is followed by a design example based on the MRF-901 transistor. The final section of the article outlines the minor performance differences between the rigorous design approach and more casual matching computations. For those readers who are interested, an appendix is provided which outlines some of the rules of vector arithmetic that are required for proper manipulation of semiconductor scattering parameters.

simple matching scheme

For any two-port device, optimum power transfer occurs when both input and output are terminated in their complex conjugate impedances. That is, for any port impedance consisting of a resistance and a series reactive component, the termination should appear as a like value of resistance in series with the *opposite* reactive component. For example, a port with an impedance of 35 + j100 ohms (35 ohms of resistance, 100 ohms of series *inductive* reactance), should be terminated in 35 - j100 ohms (35 ohms of resistance, 100 ohms of series *capacitive* reactance).

Recognizing the above, amplifier design consists of causing the ports (transistor input and output imped-

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ances) to be terminated in their complex conjugates. The problem is to identify the actual transistor input and output impedances occurring at a given frequency, and under a given set of bias conditions.

It is convenient to assume that the impedances related to the transistor's input and output reflection coefficients (s_{11} and s_{22}) approximate the device's input and output impedances. This is the approach I used in my previous articles. The fallacy (and one source of minor errors) is the fact that s_{11}, the *input reflection coefficient,* directly relates to input impedance *only* when the output is terminated in a pure resistance of 50 ohms. Similarly, s_{22}, *output reflection coefficient,* is directly related to the output impedance *only* when the input is terminated in a pure resistance of 50 ohms. In other words, varying the impedance match at one port will affect the impedance seen at the other port.

To understand the reason for this interaction, it's important to realize that no transistor is strictly a unilateral device. Any time a signal is injected into the output of a transistor amplifier, some signal will be discernible at the input. The physics of semiconductor construction allow for a feedback path which, though it may appear minimal, nontheless allows output matching to have an impact on input impedance, and vice versa.

If the input and output reflection coefficients of a transistor were both zero, this feedback path would have no effect on transistor matching. A reflection coefficient of zero indicates a corresponding port impedance of 50 ohms, nonreactive. Since s_{11} is measured with the output terminated in 50 ohms, and s_{22} is measured with the input similarly loaded, this is the only case in which terminating one port will not disrupt matching to the other. Of course, if both the input and output impedances of a transistor were 50 ohms, pure resistive, matching to a 50-ohm source and load would be considerably simplified. Unfortunately, we are not blessed with such transistors. Hence, to properly determine input and output impedances, the device's *transfer coefficients* must be considered.

S-parameters

In addition to s_{11} and s_{22}, the reflection coefficients discussed previously, a microwave transistor is characterized by a *forward transfer coefficient,* s_{21} (mathematically related to gain), and a *reverse transfer coefficient,* s_{12} (which describes the internal feedback path). Together, these four scattering parameters fully characterize the operation of the device. From them can be calculated the transistor's stability factor (tendency to oscillate under various conditions of source and load termination), maximum available gain, maximum stable gain, and equivalent input and output impedances. The s-parameters can be further manipulated to determine the device's maximum linear power output capability[3] although such an analysis is beyond the scope of this article.

It should be remembered that each of the four s-parameters varies with frequency, as well as with varying conditions of bias current and operating potential. The term "scattering" is derived from the fact that the parameters describe a set of variables, based on traveling waves incident on a port and reflected (or scattered) from it, which are evaluated with a mathematical tool called a scattering matrix.[4]

It should be further pointed out that s-parameters are *vectors.* That is, they appear as points on a Smith chart or polar plot which can be defined by both *magnitude* and *angle.* For example, at a frequency of 1.3 GHz, with

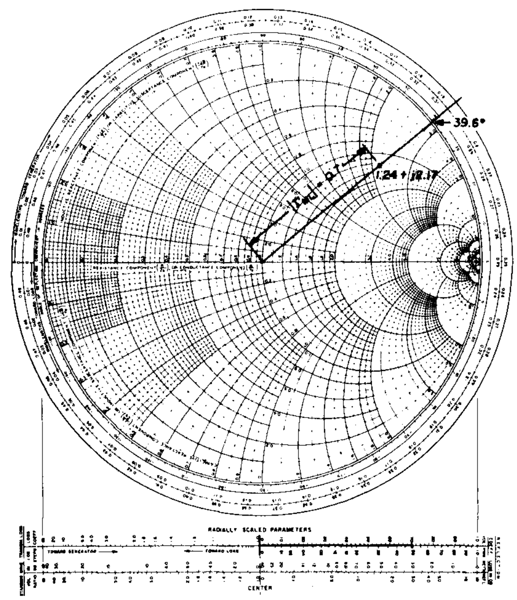

fig. 1. Using a Smith chart to plot the load impedance which exhibits the specified load reflection coefficient, 0.7 ∠ 39.6°. On this normalized Smith chart this yields 1.24 + j2.17. In a 50-ohm system the required load impedance is 62 + j108.5. To plot this point first locate the angle of the reflection coefficient on the peripheral scale and draw a line from 39.6° on this scale through the center of the chart. Referring to the radially scaled voltage reflection coefficient below the chart, measure the distance to 0.7, and transfer this length to the previously plotted line on the Smith chart. The crossover point marks the required complex load impedance.

a collector current of 10 mA and a collector-to-emitter potential of 10 volts, the common-emitter s-parameters for a Motorola MRF-901 microwave transistor are:

$$s_{11} = 0.47 \angle +161^\circ$$
$$s_{22} = 0.43 \angle -41^\circ$$
$$s_{12} = 0.08 \angle +64^\circ$$
$$s_{21} = 3.1 \angle +63^\circ$$

A complete discussion of the derivation and usefulness of the four s-parameters is available in an application note published by Hewlett-Packard.[5] Tabulations of s-parameters corresponding to various frequencies and bias conditions are available from the manufacturers of most microwave transistors.

gain and stability analysis

Before attempting to determine input and output impedances and design matching networks, it is desirable to approximate the gain capabilities of the transistor under the chosen operating conditions, and to determine whether the resulting amplifier will be stable. Three parameters which aid in such analysis are Maximum Available Gain *(MAG)*, Maximum Stable Gain *(MSG)*, and Rollett's stability factor *(K)*. K indicates the amplifier's tendency to oscillate. If K is greater than 1, the amplifier will be stable under any combination of input and output impedances or phase angles. Such an amplifier is said to be *unconditionally* stable. Conservative

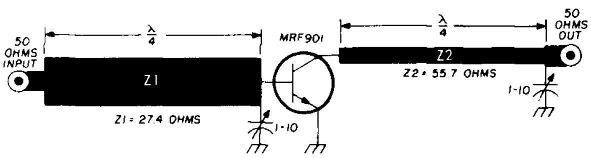

fig. 2. Basic circuit for a 1296-MHz amplifier which uses a Motorola MRF-901 transistor. Input and output matching is provided by microstriplines and 10 pF trimmer capacitors. This amplifier is unconditionally stable and gain is about 13 dB (bias networks are not shown).

design philosophy suggests that if K calculates to less than unity a different transistor or bias condition should be selected.

Maximum stable gain is, to quote WA6RDZ, "... the most important figure of merit. Transistors with high *MSG* are easy to match, easy to tune, and give high performance, trouble-free amplifiers."[6] Maximum available gain, also easily calculated, is a fairly accurate approximation of the gain you will observe in the actual circuit if it is carefully designed and built. If *MAG* is on the order of 2 or 3 dB less than *MSG*, the amplifier is likely to be both stable and reliable.

Of the above parameters, *MSG* is the most readily computed because it involves only the absolute values (magnitudes) of s_{21} and s_{12}:

$$MSG\,(dB) = 10\,log \frac{|s_{21}|}{|s_{12}|} \tag{1}$$

In order to perform the remaining calculations, the vector quantity *Δ and the scalar values B_2 are required:

$$\Delta = s_{11}\,s_{22} - s_{12}\,s_{21} \tag{2}$$

$$B_2 = 1 + |s_{22}|^2 - |s_{11}|^2 - |\Delta|^2 \tag{3}$$

It is now possible to calculate Rollett's stability factor, K

$$K = \frac{1 + |\Delta|^2 - |s_{11}|^2 - |s_{22}|^2}{2 \cdot |s_{21}| \cdot |s_{12}|} \tag{4}$$

If K proves greater than unity, go ahead and calculate maximum available gain:

$$MAG\,(dB) = MSG + 10\,log\,\left|K \pm \sqrt{K^2 - 1}\right| \tag{5}$$

where if B_2 is greater than zero (i.e., positive), the sign preceding $\sqrt{K^2 - 1}$ is negative, and if B_2 is less than zero (i.e., negative), the sign is positive. At this point in the circuit design, it is possible to determine whether the performance of the amplifier is acceptable for the intended application. If the amplifier proves only conditionally stable ($K \leqslant 1$), or if *MAG* is insufficient, select another transistor or bias point, and go through the calculations again with the new s-parameters.

*Rules for vector arithmetic are discussed in *appendix 1*.

output conjugate matching

Assuming that the gain and stability analysis indicate that the amplifier design is workable, the output circuit is designed to terminate the transistor in the complex conjugate of its *actual* output impedance. To determine the true output impedance requires a manipulation involving not only s_{22}, but also Δ and B_2 (**eqs. 2 and 3**) as well as s_{11}. To find the desired load reflection coefficient, first compute the intermediate vector quantity C_2:

$$C_2 = s_{22} - (\Delta \cdot s_{11}{}^*) \tag{6}$$

where the asterisk indicates that the complex conjugate of the immediately preceding vector is used (that is, same magnitude, angle has opposite sign).

The angle of the desired load reflection coefficient, $\Gamma_{ML\theta}$, is simply $C_{2\theta}{}^*$. The desired magnitude is found from:

$$|\Gamma_{ML}| = \frac{B_2 \pm \sqrt{B_2{}^2 - 4\,|C_2|^2}}{2\,|C_2|} \tag{7}$$

The sign preceding the radical sign is, once again, opposite to the sign on B_2. The desired load reflection coefficient may now be converted on a Smith chart into a complex impedance value, then matched to 50 ohms, as discussed in my previous articles.

input conjugate matching

Once the output load has been specified, the source reflection coefficient which will properly terminate the transistor's input is found from

$$\Gamma_{MS} = \left[s_{11} + \left(\frac{s_{12} \cdot s_{21} \cdot \Gamma_{ML}}{1 - (\Gamma_{ML} \cdot s_{22})}\right)\right]^* \tag{8}$$

where the asterisk indicates the complex conjugate (same magnitude, angle has opposite sign). This reflection coefficient may be plotted on a Smith chart to determine equivalent impedance, and the result transformed to 50 ohms.

To those readers who have Hewlett-Packard HP-45 engineering calculators, I highly recommend an article by Martin[7] which reduces formulas similar to the above to straightforward keystroke sequences. I have also published a HP-35 algorithm for series-to-parallel complex impedance conversion which may prove useful in designing matching networks.[8] Additionally, I recently derived a family of programs for the new HP-25 programmable calculator which greatly simplify all of the above calculations.*

*A complete set of HP-25 amplifier design programs is available from the author for $2.00 plus a stamped, self-addressed envelope.

design example

As noted previously, the common-emitter s-parameters for the Motorola MRF-901 transistor at 1.3 GHz, when biased at 10 volts and 10 mA, are as follows:

$$s_{11} = 0.47 \angle{+161^\circ}$$
$$s_{22} = 0.43 \angle{-41^\circ}$$
$$s_{12} = 0.08 \angle{+64^\circ}$$
$$s_{21} = 3.10 \angle{+63^\circ}$$

Using these parameters, the maximum stable gain, MSG, is calculated from **eq. 1**:

$$MSG\,(dB) = 10 \log \frac{s_{21}}{s_{12}}$$
$$= 10 \log (3.1/0.08) = 15.9\ dB$$

Before performing the remaining calculations, it's necessary to compute the vector quantity Δ (**eq. 2**) and the scalar quantity B_2 (**eq. 3**).

$$\Delta = (s_{11} \cdot s_{22}) - (s_{12} \cdot s_{21})$$
$$= [|s_{11}| \cdot |s_{22}| \angle(s_{11\theta} + s_{22\theta})] - [|s_{12}| \cdot |s_{21}| \angle(s_{21\theta} + s_{12\theta})]$$
$$= [|0.47| \cdot |0.43| \angle 161^\circ + (-41^\circ)] - [|0.08| \cdot |3.1| \angle(63^\circ + 64^\circ)]$$
$$= (0.202 \angle 120^\circ) - (0.25 \angle 127^\circ)$$

Converting to rectangular notation and subtracting the x and y components,

$$\Delta_x = 0.049 \qquad \Delta_y = -0.025$$

Returning to polar notation

$$\Delta_{tR} = \sqrt{\Delta_x{}^2 + \Delta_y{}^2} = \sqrt{0.00306} = 0.055$$
$$\Delta_{t\theta} = \arctan \Delta_y/\Delta_x = \arctan -0.500 = -26.56^\circ$$
$$\Delta = 0.055 \angle{-26.56^\circ}$$

The scalar quantity B_2 is calculated from the relationship

$$B_2 = 1 + |s_{22}|^2 - |s_{11}|^2 - |\Delta|^2$$
$$= 1 + (0.43)^2 - (0.47)^2 - (0.055)^2 = 0.96$$

With the vector quantity Δ and scalar quantity B_2 now known, it's possible to calculate Rollett's stability factor, K, from **eq. 4**.

$$K = \frac{1 + |\Delta|^2 - |s_{11}|^2 - |s_{22}|^2}{2\,|s_{21}| \cdot |s_{12}|}$$
$$= \frac{1 + (0.055)^2 - (0.47)^2 - (0.43)^2}{2(3.1)\,(0.08)} = 1.20$$

Since the stability factor is greater than 1, the amplifier will be unconditionally stable.

The maximum available gain, MAG, of the amplifier is calculated with **eq. 5**.

$$MAG\,(dB) = MSG\,(dB) + 10 \log |K \pm \sqrt{K^2 - 1}|$$

Since B_2 is greater than zero (i.e., positive) the sign of the radical is minus.

$$MAG\,(dB) = 15.9\ dB + 10 \log |1.20 - \sqrt{1.20^2 - 1}|$$
$$= 15.9\ dB + 10 \log 0.54$$
$$= 15.9 + (-2.7) = 13.2\ dB$$

Since MAG is approximately 3 dB lower than MSG, the amplifier can be expected to tune easily.

output matching. To terminate the transistor in the complex conjugate of its output impedance, first compute the intermediate vector quantity C_2 from **eq. 6**.

$$C_2 = s_{22} - (\Delta \cdot s_{11}{}^*)$$

remembering that the angle of $s_{11}{}^*$ has a sign opposite to that of s_{11} (in this case, $s_{11}{}^* = 0.47 \angle{-161^\circ}$

$$C_2 = (0.43 \angle{-41^\circ}) - [(0.054 \angle{-25.61^\circ}) \cdot (0.47 \angle{-161^\circ})]$$
$$= (0.43 \angle{-41^\circ}) - (0.02 \angle 173.4^\circ)$$

Converting to rectangular notation and subtracting the x and y components,

$$C_{2x} = 0.344 \qquad C_{2y} = -0.284$$

Returning to polar notation

$$C_{2R} = \sqrt{C_{2x}{}^2 + C_{2y}{}^2} = \sqrt{0.199} = 0.45$$
$$C_{2\theta} = \arctan C_{2y}/C_{2x} = \arctan -0.826 = -39.6^\circ$$
$$C_2 = 0.45 \angle{-39.6^\circ}$$

The angle of the load reflection coefficient, $\Gamma_{ML\theta}$ is $C_{2\theta}{}^*$ where the asterisk indicates that the sign of the angle is changed. In this case, $C_{2\theta}{}^* = +39.6^\circ$. The magnitude of the load reflection coefficient is found from **eq. 7**:

$$|\Gamma_{ML}| = \frac{B_2 \pm \sqrt{B_2{}^2 - 4|C_2|^2}}{2|C_2|}$$

where the sign ahead of the radical is opposite to the sign on B_2.

$$|\Gamma_{ML}| = \frac{0.96 - \sqrt{(0.96)^2 - 4(0.45)^2}}{2(0.45)}$$
$$= 0.70$$
$$\Gamma_{ML} = 0.70 \angle 39.6^\circ$$

This quantity may be plotted on a Smith chart to determine the desired load impedance as shown in **fig. 1.*** This yields

$$Z_L = 62.0 + j108.5$$

*The load impedance can also be calculated from the relationship

$$Z_L = Z_o \frac{1 + \Gamma_{ML}}{1 - \Gamma_{ML}}$$

where Z_L is the complex load impedance $(R \pm jX)$, $Z_o = 50 + j0$, and Γ_{ML} is the complex load reflection coefficient. Since all quantities are complex numbers, vector arithmetic is required.

Input matching. Now that the output load impedance has been specified, the source reflection coefficient which will properly terminate the input to the transistor can be calculated from **eq. 8**:

$$\Gamma_{MS} = \left[s_{11} + \left(\frac{s_{12} \cdot s_{21} \cdot \Gamma_{ML}}{1 - (\Gamma_{ML} \cdot s_{22})}\right)\right]*$$

$$= (0.47 \angle 161^\circ) + \left(\frac{(0.08 \angle 64^\circ)(3.1 \angle 63^\circ)(0.7 \angle 39.6^\circ)}{1 - [(0.7 \angle 39.6^\circ)(0.43 \angle -41^\circ)]}\right)*$$

$$= (0.47 \angle 161^\circ) + \left(\frac{(0.08 \cdot 3.1 \cdot 0.7) \angle 64^\circ + 63^\circ + 39.6^\circ}{1 - [(0.7 \cdot 0.43) \angle (39.6^\circ - 41^\circ)]}\right)*$$

$$= (0.47 \angle 161^\circ) + \left(\frac{0.17 \angle 166.6^\circ}{0.70 \angle 0.60^\circ}\right)*$$

$$= [(0.47 \angle 161^\circ) + (0.24 \angle 166^\circ)]*$$

Converting to rectangular notation and adding the x and y components,

$$\Gamma_{MSx} = -0.68 \qquad I_{MSy} = 0.21$$

Returning to polar notation

$$\Gamma_{MSR} = \sqrt{\Gamma_{MSx}^2 + \Gamma_{MSy}^2} = \sqrt{0.51} = 0.71$$

$$\Gamma_{MS\theta} = \arctan \Gamma_{MSy}/\Gamma_{MSx} = \arctan -0.31$$

$$= 162.7^\circ$$

$$\Gamma_{MS} = 0.71 \angle 162.7^\circ * = 0.71 \angle -162.7^\circ$$

The source reflection coefficient for a complex conjugate input match may be plotted on a Smith chart to determine the corresponding source impedance. This yields the series complex impedance, $Z_s = 8.7 - j7.4$ ohms (parallel complex impedance, $Z_p = 15 \| -j17.6$ *ohms*).

Matching networks. An input conjugate match can be obtained by shunting the transistor base with a capacitive reactance equal to the desired parallel equivalent reactance value (-j17.6 ohms) and transforming the source impedance to the required parallel resistance value (15 ohms) through a quarter-wavelength transmission line. The required capacitance value is found from the familiar reactance equation

$$C = \frac{1}{2\pi f X_c}$$

At 1296 MHz:

$$C = \frac{1}{2\pi(1296 \cdot 10^6)\, 17.6} = 6.98\ pF$$

A 10 pF trimmer capacitor will assure a proper reactive termination.

Transformation of the resistive component (to a 50-ohm source in this case) is accomplished with a quarter-wavelength transmission line which has a characteristic impedance Z_o, equal to the geometric mean of the source impedance Z_f, and the parallel input resistance, R_p.

$$Z_o = \sqrt{R_p \cdot Z_f}$$

With a 50-ohm source and a parallel input resistance of 15 ohms

$$Z_o = \sqrt{15 \cdot 50} = 27.4\ ohms$$

At 1296 MHz this is easily provided by a microstrip transmission line 0.26 inches (6.5mm) wide and 1.16 inches (29.5mm) long on a 1/16-inch (1.5mm) double-clad, fiberglass printed-circuit board.

A similar quarter-wavelength transformer can be designed to match the resistive component, R_s, of the complex series output impedance (62 ohms) to a 50-ohm termination, Z_t. As before, the characteristic impedance of the transmission line is given by

$$Z_o = \sqrt{R_s \cdot Z_t} = \sqrt{62 \cdot 50} = 55.7\ ohms$$

At 1296 MHz this is provided by a microstrip transmission line 0.09 inches (2.3mm) wide and 1.21 inches (30.7mm) long on 1/16-inch (1.5mm) double-clad, fiberglass printed-circuit board.

The required inductive reactance in series with the collector (+j108.5 ohms) is provided by shunting a capacitive reactance across the output end of the quarter-wavelength transformer. The required capacitive reactance is given by

$$X_c = \frac{Z_o^2}{X_s} = \frac{55.7^2}{108.5} = 28.6\ ohms$$

At 1296 MHz:

$$C = \frac{1}{2\pi(1296 \cdot 10^6)\, 28.6} = 4.3\ pF$$

Again, a 10 pF trimmer capacitor will suffice. The circuit in **fig. 2** shows the complete matching layout.

performance comparison

A simplified amplifier design, in which source and load impedances appear as the complex conjugate of the impedances related to s_{11} and s_{22}, yields the circuit shown in **fig. 3**. This circuit was derived by matching to the following assumed shunt-equivalent impedances:

Parallel: Z_{in} *(derived from* s_{11}*)*

$= 21 \| j56.5$ *ohms*

Series: Z_{out} *(derived from* s_{22}*)*

$= 75 - j52.5$ *ohms*

A more rigorous analysis shows the actual device impedances to be:

Parallel: Z_{in} *(actual)* $= 15 \| +j17.6$ *ohms*

Series: Z_{out} *(actual* $= 62.0 - j108.5$ *ohms*

Note that the reactive components of the shunt input impedance and the series output impedance differ significantly. Thus some degree of mismatch can be anticipated if the circuit of **fig. 3** is built as shown. Since the actual device impedances are now known, this mismatch can be accurately predicted.

As it happens, only the resistive component of the transistor's input or output complex impedance sees a mismatch. This is because the tuning range of the trimmer capacitors in **fig. 3** is sufficiently wide to properly terminate the reactive components. The input and out-

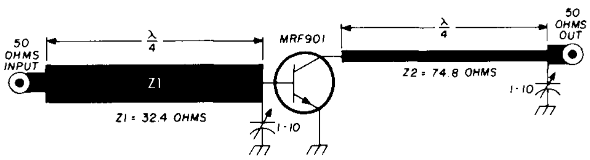

fig. 3. 1296-MHz amplifier which was designed with a simplified method. Input and output matching is provided by microstriplines and 10 pF trimmer capacitors. This amplifier is unconditionally stable and has about 0.5 dB less gain than the circuit shown in fig. 2 (bias networks are not shown).

put mismatches are determined by transforming the actual resistive components through the existing quarter-wave length transformers and comparing the resulting impedance to 50 ohms. Referring to **fig. 3,**

$$Z_{in}\ (amplifier) = \frac{Z_1^{\ 2}}{R_{p\ in}} = \frac{32.4^2}{15.0} = 70.0\ ohms$$

$$Z_{out}\ (amplifier) = \frac{Z_1^{\ 2}}{R_{s\ out}} = \frac{74.8^2}{62.0} = 90.2\ ohms$$

Thus, the input vswr is 1.4:1 and the output vswr is 1.8:1, calculated values which correlate quite closely with those values observed in the actual amplifiers.

These input and output mismatches will result in somewhat lower stage gain than available from a properly terminated device Actual stage gain is found from

$$A_p\ (dB) = MAG + G_1 + G_2$$

where G_1 and G_2 are both negative and represent the mismatch losses at the input and output, respectively. Since G_1 (for a 1.4:1 vswr) is about -0.1 dB, and G_2 (for a 1.8:1 vswr) is about -0.4 dB,

$$A_p = 13.2 + (-0.1) + (-0.4)$$
$$= 12.7\ dB$$

This closely represents the measured gain of the amplifier shown in **fig. 3.**

summary

A method has been outlined for using device s-parameters to analyze the gain and stability of a microwave amplifier, and to determine appropriate source and load impedances for a complex conjugate match. It has been shown that designing around the reflection coefficients of a particular transistor (while ignoring the transfer coefficients) resulted in input and output mismatches of 1.4:1 and 1.8:1, respectively, while degrading overall amplifier gain by approximately 0.5 dB. This is a modest penalty for enjoying the convenience of a simplistic design approach. Whether the additional performance available from the more rigorous design method is justified depends largely upon the goals of the designer, and the intended application of the amplifier.

references

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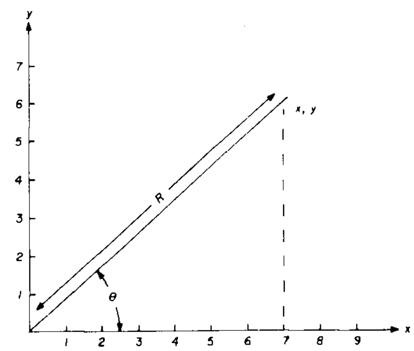
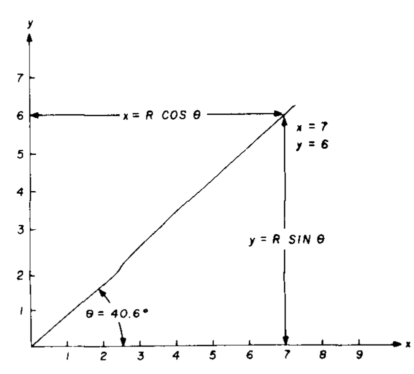
appendix 1

vector arithmetic

In the application of s-parameter design equations, it's necessary to perform numerous computations involving vector quantities. Vectors may be expressed either in conventional polar notation (magnitude *R* and associated angle *θ*) or may be resolved into their rectangular



components (*x* and *y* displacement on Cartesian coordinates) as shown below. Since the vector is part of a right triangle, manipulation between the two forms of notation involves the application of trigonometric functions. These manipulations may be accomplished on a slide rule, manually with the aid of trig tables, or on a hand-held digital calculator.



Readers who have advanced scientific calculators which include polar-rectangular conversion and summation keys will find the process considerably simplified. The following review is for the benefit of those not so fortunate.

1. Resolving vectors. Vector arithmetic often requires that the x and y components of the vector be known. Any vector, V, described by magnitude, R, and angle θ, can be resolved into its x and y components with the following formulas

$$x = R\cos\theta$$
$$y = R\sin\theta$$

Example: What are the x and y components of the vector $9.22\angle 40.6^\circ$ $(R = 9.22, \theta = 40.6^\circ)$?

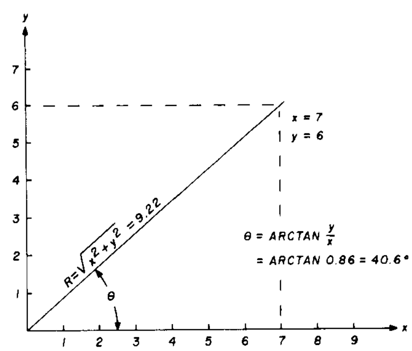
$$x = R\cos\theta = 9.22\cos 40.6^\circ$$
$$= 9.22 \cdot 0.76 = 7.00$$
$$y = R\sin\theta = 9.22\sin 40.6^\circ$$
$$= 9.22 \cdot 0.65 = 6.00$$

2. Constructing vectors. The results of vector addition and subtraction (reviewed later) generally appear as x and y components of the resultant vector. These coordinates can be converted to a polar vector of magnitude, R, and angle, θ, with measurements on a graphical plot, or by using a trigonometric solution. Since the vector is represented by the hypotenuse of a right triangle formed by dimensions x and y, the Pythagorean theorem may be used to find the magnitude R:

$$R = \sqrt{x^2 + y^2}$$

Trigonometry is used to calculate the angle θ

$$\theta = \arctan y/x$$



Example: What is the magnitude, R, and angle θ, of the vector described by the Cartesian coordinates, $x = 7$, $y = 6$?

$$R = \sqrt{x^2 + y^2} = \sqrt{49 + 36}$$
$$= \sqrt{85} = 9.22$$
$$\theta = \arctan y/x = \arctan 6/7$$
$$= \arctan 0.86 = 40.6^\circ$$

3. Vector addition. Any two vectors V_1 and V_2, which have been resolved into x and y components V_{1x}, V_{1y}, V_{2x}, and V_{2y}, may be added by summing the respective x and y components. The summed components may then be constructed into a resultant vector V_t, of magnitude V_{tR} and angle $V_{t\theta}$.

Example: What is the sum of vectors V_1 and V_2 when

$$V_1 = 1.5\angle +40^\circ$$
$$V_2 = 2.0\angle -60^\circ$$
$$V_{1x} = V_{1R}\cos V_{1\theta} = 1.5\cos 40^\circ = 1.15$$
$$V_{1y} = V_{1R}\sin V_{1\theta} = 1.5\sin 40^\circ = 0.96$$
$$V_{2x} = V_{2R}\cos V_{2\theta} = 2.0\cos -60^\circ = 1.00$$
$$V_{2y} = V_{2R}\sin V_{2\theta} = 2.0\sin -60^\circ = -1.73$$
$$\Sigma_x = V_{1x} + V_{2x} = 1.15 + 1.00 = 2.15$$
$$\Sigma_y = V_{1y} + V_{2y} = 0.96 + (-1.73) = -0.77$$
$$V_{tR} = \sqrt{\Sigma_x{}^2 + \Sigma_y{}^2} = \sqrt{2.15^2 + (-0.77)^2}$$
$$= \sqrt{5.22} = 2.28$$
$$V_{t\theta} = \arctan(\Sigma_y/\Sigma x) = \arctan(-0.77/2.15)$$
$$= -19.70^\circ$$
$$V_{t\theta} = V_1 + V_2 = 2.28\angle -19.70^\circ$$

4. Vector subtraction. Any two vectors V_1 and V_2, which have been resolved into x and y components V_{1x}, V_{1y}, V_{2x}, and V_{2y}, may be subtracted one from the other by subtracting their respective x and y components. The results of such subtraction comprise the x and y components of the resulting vector V_t, which may be constructed to yield magnitude V_{tR} and angle $V_{t\theta}$.

Example: What is the difference when vector V_2 is subtracted from vector V_1?

$$V_1 = 0.8\angle -40^\circ$$
$$V_2 = 0.4\angle +120^\circ$$
$$V_{1x} = V_{1R}\cos V_{1\theta} = 0.8\cos -40^\circ = +0.61$$
$$V_{1y} = V_{1R}\sin V_{1\theta} = 0.8\sin -40^\circ = -0.51$$
$$V_{2x} = V_{2R}\cos V_{2\theta} = 0.4\cos 120^\circ = -0.20$$
$$V_{2y} = V_{2R}\sin V_{2\theta} = 0.4\sin 120^\circ = +0.35$$
$$(\Sigma_{-x}) = V_{1x} - V_{2x} = +0.61 - (-0.20) = +0.81$$
$$(\Sigma_{-y}) = V_{1y} - V_{2y} = -0.51 - (+0.35) = -0.86$$
$$V_{tR} = \sqrt{(\Sigma_{-x})^2 + (\Sigma_{-y})^2} = \sqrt{(0.81)^2 + (-0.86)^2}$$
$$= \sqrt{1.40} = 1.18$$
$$V_{t\theta} = \arctan(\Sigma_{-y})/(\Sigma_{-x}) = \arctan(-0.86/+0.81)$$
$$= -46.7^\circ$$
$$V_t = V_1 - V_2 = 1.18\angle -46.7^\circ$$

5. Vector multiplication. For any two vectors, V_1 and V_2, each described by a magnitude, R, and an angle, θ, the vector product is found by multiplying the the magnitudes and adding the angles:

$$V_{tR} = V_{1R} \times V_{2R}$$
$$V_{t\theta} = V_{1\theta} + V_{2\theta}$$

Example: What is the vector product of V_1 and V_2 when

$$V_1 = 0.8\angle 45^\circ$$
$$V_2 = 0.65\angle -118^\circ$$
$$V_{tR} = V_{1R} \times V_{2R} = 0.8 \times 0.65 = 0.52$$
$$V_{t\theta} = V_{1\theta} + V_{2\theta} = +45^\circ + (-118^\circ) = -73^\circ$$
$$V_t = V_1 \cdot V_2 = 0.52\angle -73^\circ$$

6. Vector division. For any two vectors, V_1 and V_2, each described by a magnitude, R, and an angle, θ, the vector quotient is found by dividing the magnitudes and subtracting the angles:

$$V_{tR} = V_{1R} \div V_{2R}$$
$$V_{t\theta} = V_{1\theta} - V_{2\theta}$$

Example: What is the vector quotient when V_1 is divided by vector V_2?

$$V_1 = 0.96\angle +64^\circ$$
$$V_2 = 0.42\angle -102^\circ$$
$$V_{tR} = V_{1R} \div V_{2R} = 0.96 \div 0.42 = 2.29$$
$$V_{t\theta} = V_{1\theta} - V_{2\theta} = +64^\circ - (-102^\circ) = +166^\circ$$
$$V_t = 2.29\angle +166^\circ$$

7. Maximum angle. Whenever a vector manipulation yields an expression whose angle exceeds ±180°, subtract the absolute value of the angle from 360°, and assign to the resulting angle a sign opposite to that of the original angle.

Examples: $-196^\circ = +360 - |-196|$

$$= +360 - 196 = +164^\circ$$

$$265^\circ = -360 + |+265|$$

$$= -360 + 265 = -95^\circ$$

8. **Compound expressions.** In expressions involving both vector and scalar quantities, treat the scalar quantity as though it were a vector of angle 0°.

Example: In the expression for the source reflection coefficient which will properly terminate the transistor's input (**eq. 8**), the product of the load reflection coefficient, Γ_{ML}, and s_{22} are subtracted from one. If $(\Gamma_{ML} \cdot s_{22}) = 0.65 \angle -118^\circ$, what is the value of the expression, $1 - (\Gamma_{ML} \cdot s_{22})$?

$$V_1 = 1 \angle 0^\circ$$

$$V_2 = 0.65 \angle -118^\circ$$

$$V_{1x} = V_{1R} \cos V_{1\theta} = 1 \cos 0^\circ = 1.00$$

$$V_{1y} = V_{1R} \sin V_{1\theta} = 1 \sin 0^\circ = 0$$

$$V_{2x} = V_{2R} \cos V_{2\theta} = 0.65 \cos -118^\circ = -0.31$$

$$V_{2y} = V_{2R} \sin V_{2\theta} = 0.65 \sin -118^\circ = -0.57$$

$$\Sigma_{-x} = V_{1x} - V_{2x} = 1.00 - (-0.31) = 1.31$$

$$\Sigma_{-y} = V_{1y} - V_{2y} = 0 - (-0.57) = 0.57$$

$$V_{1R} = \sqrt{(\Sigma_{-x})^2 + (\Sigma_{-y})^2} = \sqrt{1.31^2 + 0.57^2}$$

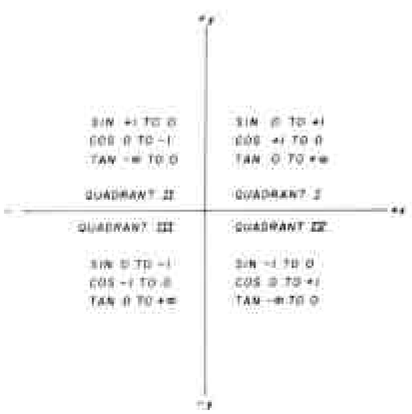
$$= \sqrt{2.04} = 1.43$$

$$V_{1\theta} = \arctan(\Sigma_{-y}/\Sigma_{-x}) = \arctan 0.57/1.31$$

$$= 23.51^\circ$$

$$1 - (\Gamma_{ML} \cdot s_{22}) = 1.43 \angle 23.51^\circ$$

9. **Angular functions.** Since most trigonometry tables show only the functions to +90°, when working with vectors which may fall in any of the four quadrants below (0 through 360 degrees) this can lead to ambiguities in specifying the angle θ of a resultant vector. Note that the



tangent function varies from zero to +∞, from zero to 90 degrees, from −∞ to zero in the second quadrant (90 to 180 degrees), from zero to +∞ in the third quadrant (180 to 270 degrees), and from −∞ to zero in the fourth quadrant (270 through 360 degrees). The sine and cosine functions are also ambiguous, as shown, but this doesn't create a problem in vector arithmetic.

In the expression for the angle of the source reflection coefficient, $\Gamma_{MS\theta}$, in the design example $\Gamma_{MSx} = -0.68$ and $\Gamma_{MSy} = 0.21$. Therefore,

$$\Gamma_{MS\theta} = \arctan -0.68/0.21 = \arctan 0.31$$

$$= -17.3^\circ$$

This is in the fourth quadrant whereas the x and y values place the vector in the second quadrant. Therefore, the correct value for the angle is $180^\circ + (-17.3^\circ) = 162.7^\circ$. The same sort of ambiguity exists for the first and third quadrants, and can only be resolved by inspection.

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